Figures of Merit

- Frequency
- Tuning range (Desired channels, Process variation)
- Phase noise / Jitter
- Spurious performance (Spur)
- Acquisition / Settling time
- Capture range
- Lock range
- Stability
- Power consumption
- Supply voltage

Phase-Locked Loop

- VCO generates oscillation frequency $f_o$
- Phase detector (PD) compares phases of $f_{ref}$ and $f_o$ and produces an output whose average value is proportional to phase difference
- LPF filters high-frequency components from PD output to obtain average value to control VCO
**PLL – Linear Model**

\[ \phi_{ref} \rightarrow K_{PD} \rightarrow G_{LPF}(s) \rightarrow \frac{K_{VCO}}{s} \rightarrow \phi_{VCO} \]

\[ \phi_{out} \rightarrow \phi_{out} \]

**Phase Transfer Functions**

\[
H(s)_{\text{open}} = \left. \frac{\phi_{out}(s)}{\phi_{ref}(s)} \right|_{\text{open}} = K_{PD}G_{LPF}(s)\frac{K_{VCO}}{s}
\]

\[
H(s)_{\text{close}} = \left. \frac{\phi_{out}(s)}{\phi_{ref}(s)} \right|_{\text{close}} = \frac{H(s)_{\text{open}}}{1 + H(s)_{\text{open}}}
\]

\[
= \frac{K_{PD}G_{LPF}(s)K_{VCO}}{s + K_{PD}G_{LPF}(s)K_{VCO}}
\]

**Phase Transfer Function – First-Order Loop Filter**

\[
G_{LPF}(s) = \frac{1}{1 + \frac{s}{\omega_{LPF}}}
\]

\[
H_{ref}(s)_{\text{close}} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}
\]

\[
\therefore \quad \omega_n = \sqrt{\omega_{LPF}K_{PD}K_{VCO}}
\]

\[
\therefore \quad \zeta = \frac{1}{2} \sqrt{\frac{\omega_{LPF}}{K_{PD}K_{VCO}}} = \frac{\omega_n}{2K_{PD}K_{VCO}}
\]

**Phase Transfer Function – First-Order Loop Filter**

- Exhibits a second-order transfer function: a pole from VCO and another from LPF
- Damping factor is inversely proportional to the loop gain, which is not desirable
- Typically, for maximally flat frequency response, damping factor \( \zeta = 0.707 \)
- \( \omega_n \) cannot be maximized independently with loop gain \( K = K_{PD}K_{VCO} \) given optimal value of \( \zeta \)
- Transfer function to reference is a low-pass response \( \Rightarrow \) high-frequency reference phase noise will be attenuated \( \Rightarrow \) want small loop bandwidth!
Phase Transfer Function – First-Order Loop Filter

- Transfer function to reference is a low-pass response $\Rightarrow$ high-frequency variation (phase noise) at reference will be attenuated and cause small change in output frequency variation $\Rightarrow$ want small loop bandwidth!

- On the other hand, low-frequency phase noise will be transferred to the output directly without modification

$$H_{error}(s) = \frac{\Phi_{error}(s)}{\Phi_{ref}(s)} = \frac{\Phi_{ref}(s) - \Phi_{out}(s)}{\Phi_{ref}(s)}$$

$$1 - H_{ref}(s) = \frac{s^2 + 2\zeta\omega_n s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\therefore H_{error}(s) \to 0$ as $s \to 0$

Phase Transfer Function – Step Response for First-Order Loop Filter

$$\Phi_{ref}(s) = \Delta \omega * u(t)$$

$$\therefore \Phi_{error}(s) = H_{error}(s)\Phi_{ref}(s) = \frac{s^2 + 2\zeta\omega_n s}{s^2 + 2\zeta\omega_n s + \omega_n^2} \cdot \Delta \omega$$

$$\therefore \Phi_{error}(t)_{|t\to\infty} = \lim_{s \to 0} [s \Phi_{error}(s)] = 2\zeta \frac{\Delta \omega}{\omega_n} = \frac{\Delta \omega}{K_{PD} K_{VCO}}$$

$\Rightarrow$ Large loop gain is desirable for small phase error

VCO’s Phase Transfer Function

$$H_{vco}(s) = \frac{\phi_{out}(s)}{\phi_{vco}(s)} = \frac{1}{1 + H(s)}$$

$$= \frac{s}{s + K_{PD} G_{LPF}(s) K_{VCO}} = \frac{s^2 + 2\zeta\omega_n s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- Transfer function to VCO is a high-pass response $\Rightarrow$ Low-frequency VCO phase noise will be attenuated $\Rightarrow$ Want large loop bandwidth
Settling Time and Stability

- Loop bandwidth is defined as unity-gain frequency of open-loop gain
- For fast switching, need large loop bandwidth and large natural frequency
- For stability, loop bandwidth << reference and phase margin > 45 degrees

Loop Bandwidth and Phase Margin [Brennan]

\[
H(s)\big|_{open} = K_{PD}G_{LPF}(s)\frac{K_{VCO}}{s} = \frac{2\zeta\omega_n s + \omega_n^2}{s^2}
\]

\[
|H(s)|\omega=\omega_{BW} = \sqrt{1 + 4\zeta^2} \frac{(\omega_{BW} / \omega_n)^2}{(\omega_{BW} / \omega_n)^2} \equiv 1
\]

Settling Time and Loop Bandwidth

\[
t_s \propto \frac{1}{\zeta \omega_n} \approx \frac{1}{\omega_{BW}}
\]

\[
\omega_{BW} \ll \frac{\omega_{ref}}{10}
\]
Design Trade-Off

- Smaller loop bandwidth
  - More attenuation of phase noise from PD, LPF => Better spur performance
  - For stability: loop bandwidth/reference < 1/10
  - For better noise suppression: ratio ~ 1/100
- Larger loop bandwidth
  - Faster switching time
  - Smaller component values and chip area
  - More attenuation of phase noise from VCO
  - May cause instability unless reference is increased

Charge-Pump Phase-Locked Loop

- Employs a phase-frequency detector (PFD) and a charge pump (CP) instead of a simple phase detector (PD)

Charge-Pump Phase-Locked Loop

- Transfer function of PFD and charge pump has a pole at zero (Integrator)
- Together with another pole at zero from VCO, PLL system may be unstable
- To ensure stability, a zero is added by including resistor $R_z$ in series with $C_z$
- In practice, a capacitor $C_P$ is connected in parallel to suppress ripples at $V_C$, which would introduce a third pole
  ⇒ Third-order, type-2 PLL

\[
K_{PFD} = \frac{I_{CP}}{2\pi}; \quad G_{LPF}(s) = \left( R_z + \frac{1}{C_z s} \right)
\]

\[
H(s)\bigg|_{open} = \frac{I_{CP} K_{VCO}}{2\pi} \frac{1 + s \tau_z}{s^2 C Z} \quad \therefore \omega_C = \frac{I_{CP} K_{VCO} R_Z}{2\pi}
\]

\[
H_{ref}(s)\bigg|_{close} = \frac{K_{PFD}(s) G_{LPF}(s) K_{VCO}}{s + K_{PFD}(s) G_{LPF}(s) K_{VCO}}
\]

\[
= H_o \frac{(s + \omega_z)}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]

\[
C_{P} = 0, 2^{nd}-Order
\]
Charge-Pump Phase-Locked Loop

(CP = 0, 2nd-Order)

\[ H_o = \frac{I_{cp} R_z}{2\pi} K_{vco} \]
\[ \omega_z = -\frac{1}{R_z C_z} \]
\[ \omega_n = \sqrt{\frac{I_{cp}}{2\pi C_z} K_{vco}} \neq f(R_z) \]
\[ \zeta = \frac{R_z}{2} \sqrt{\frac{I_{cp} C_z}{2\pi} K_{vco}} = \frac{\omega_n R C_z}{2} = \frac{\omega_n}{2\omega_z} \]
\[ \zeta \omega_n = \frac{I_{cp} R_z}{4\pi} K_{vco} \neq f(C_z) \]

Charge-Pump Phase-Locked Loop

(with CP, 3rd-Order)

\[ K_{PFD} = \frac{I_{cp}}{2\pi} \]
\[ G_{LPF}(s) = \frac{1+s \tau_z}{s(C_z+C_p)(1+s \tau_P)} \]
\[ \tau_z = R_z C_z \]
\[ \tau_P = R_z (C_z // C_p) \]
\[ H(s)|_{open} = \frac{I_{cp} K_{vco}}{2\pi} \frac{1+s \tau_z}{s^2 (C_z+C_p)(1+s \tau_P)} \]
\[ \omega_c = \frac{I_{cp} K_{vco} R_z}{2\pi} \frac{C_z}{(C_z+C_p)} \]

Charge-Pump Phase-Locked Loop

- Natural frequency \( \omega_n \) does not depend on \( R_z \)
- Both \( \omega_n \) and \( \zeta \) can be maximized simultaneously by increasing \( I_{cp} \) or \( K_{vco} \)
- Time constant \( (\zeta \omega_n) \) and settling time do NOT depend on \( C_z \)
Charge-Pump Phase-Locked Loop (with $C_P$, 3\textsuperscript{rd}-Order)

- Extra pole by $C_P$ should be placed far to the right from the cross-over frequency $\omega_c$
- Typically, for stability and good phase margin, the zero can be 3-4 times smaller than the cross-over frequency, which in turn is 3-4 times smaller than the extra 3\textsuperscript{rd} pole
- On the other hand, $C_P$ needs to be large enough to minimize $kT/C$ noise
- As a result, $C_Z$ needs to be even much larger ($\sim 10nF$) => needs to be put off-chip => dual-path loop filter can be used to minimize capacitance
Synthesizer’s Phase Transfer Function (from Reference) – First-Order Loop Filter

\[ G_{LPF}(s) = \frac{1}{1 + \frac{s}{\omega_{LPF}}} \]

\[ H_{ref}(s)|_{close} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \]

\[ \omega_n = \sqrt{\omega_{LPF} K_{pD} K_{VCO} / M} \]

\[ \zeta = \frac{1}{2} \sqrt{\frac{\omega_{LPF} M}{K_{pD} K_{VCO}}} = \frac{\omega_n}{2K_{pD} K_{VCO}} \]

- Exhibits a second-order transfer function: a pole from VCO and another from LPF
- \( \omega_n \) cannot be maximized by loop gain independently of \( \zeta \)
- Transfer function to reference is a low-pass response \( \Rightarrow \) High-frequency reference phase noise will be attenuated \( \Rightarrow \) Want small loop bandwidth
- Noise would be reduced with smaller division modulus M

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Synthesizer Using Charge-Pump Phase-Locked Loop (CPPPL)

\[ K_{PFD} = \frac{I_{CP}}{2\pi}; \quad G_{LPF}(s) = \left( R_z + \frac{1}{C_z s} \right) \]

\[ H_{ref}(s)|_{close} = \frac{MK_{PFD}(s)G_{LPF}(s)K_{VCO}}{Ms + K_{PFD}(s)G_{LPF}(s)K_{VCO}} \]

\[ H^o = \frac{(s + \omega_z)}{s^2 + 2\zeta\omega_n s + \omega_n^2} \]

- \( H^o = \frac{I_{CP} R_z K_{VCO}}{2\pi}; \quad \omega_z = -\frac{1}{R_z C_z} \)
- \( \omega_z = \sqrt{\frac{I_{CP} K_{VCO}}{2\pi C_z M}} \)
- \( \zeta = \frac{R_z}{2} \left( \frac{I_{CP} C_z K_{VCO}}{M} \right) = \frac{\omega_z R_z C_z}{2} = \frac{\omega_n}{2\omega_z} \)
- \( \zeta\omega_z = \frac{I_{CP} R_z K_{VCO}}{4\pi M} \)
References