Lecture 7: Optical couplers

- Directional couplers
- Coupling coefficient
- Symmetric directional couplers
- Grating waveguide couplers
- Distributed Bragg reflectors
- Surface input and output couplers

References: This lecture follows the materials from Photonic Devices, Jia-Ming Liu, Chapter 5.
Optical couplers

- Optical couplers are passive devices that couple light through waveguides or fibers.
- They play a very important role in the applications of photonic devices and systems.
- Optical couplers are used in many applications including the interface between devices in a system or can be important devices themselves.
- The most straightforward application is to route optical waves around for coupling different devices.
- Sophisticated applications include devices such as polarization converters, mode converters, guided-wave beam splitters, beam combiners, directional couplers, wavelength filters, wavelength multiplexers, and so on.
Directional couplers
Directional couplers

- Directional couplers are multiple-waveguide couplers used for *codirectional* coupling.
- They can be used in many applications, including power splitters, optical switches, wavelength filters and polarization selectors.
- Here we consider *two-channel directional couplers*, which consist of two parallel singlemode waveguides, i.e. each waveguide supports only its fundamental mode.
Directional couplers

Coupling between the two single-mode waveguides in such a two-channel directional coupler is described by

\[
\frac{d\tilde{A}}{dz} = i\kappa_{ab}\tilde{B}\exp i2\delta z
\]

\[
\frac{d\tilde{B}}{dz} = i\kappa_{ba}\tilde{A}\exp -i2\delta z
\]

\[2\delta = (\beta_b + \kappa_{bb}) - (\beta_a + \kappa_{aa})\]

\[c_{aa} = c_{bb} = 1\]
Directional couplers

- Using $c_{aa} = c_{bb} = 1$ for evaluation of the coupling coefficients, we have:

  \[ \kappa_{aa} = \frac{\tilde{K}_{aa} - c_{ab} \tilde{K}_{ba}}{1 - c_{ab} c_{ba}} \]
  \[ \kappa_{bb} = \frac{\tilde{K}_{bb} - c_{ba} \tilde{K}_{ab}}{1 - c_{ab} c_{ba}} \]
  \[ \kappa_{ab} = \frac{\tilde{K}_{ab} - c_{ab} \tilde{K}_{ba}}{1 - c_{ab} c_{ba}} \]
  \[ \kappa_{ba} = \frac{\tilde{K}_{ba} - c_{ba} \tilde{K}_{ab}}{1 - c_{ab} c_{ba}} \]

  because the waves in both waveguides are forward propagating.

- In general, the two waveguides are not necessarily identical.

- Then the directional coupler is not symmetric, and $\kappa_{ab} \neq \kappa_{ba}^*$.

- If the two waveguides are identical, the directional coupler is symmetric. Then, $\kappa_{ab} = \kappa_{ba}^*$, $\kappa_{aa} = \kappa_{bb}$, and $\beta_a = \beta_b$ (i.e. the directional coupler is perfectly phase matched with $2\delta = 0$).
Coupling coefficient
Coupling coefficient

- The coefficients are more complicated than those in the coupling between two modes in the same waveguide because of the existence of the overlap coefficient and the fact that $\kappa_{ab} \neq \kappa_{ba}^*$ in general.
- Many parameters have to be calculated in order to obtain the coupling coefficients $\kappa_{ab}$ and $\kappa_{ba}$.
- Here, we consider a simple example, namely, the two-channel directional coupler with step-index waveguides on the same substrate.
- We assume that the waveguides are planar slab waveguides for simplicity although practical directional couplers are often made of channel waveguides.
- We also consider only isotropic waveguides where TE and TM modes do not couple.
Coupling coefficient

- The two waveguides have widths $d_a$ and $d_b$ and guiding-layer refractive indices $n_a$ and $n_b$.
- They are separated by a distance $s$ between the two near edges of the guiding layers. The index of refraction of the substrate is $n_2$.
- When $n_a = n_b = n_1$ and $d_a = d_b = d$, the coupler is symmetric. Otherwise, it is asymmetric.
- To calculate the relevant coefficients, we first identify the perturbation $\Delta \varepsilon$ for each waveguide.
- For waveguide $a$, the susceptibility step of waveguide $b$ above the substrate is the perturbation, and vice versa.
Therefore, we have

\[ \Delta \varepsilon_a = \varepsilon_0 (n_b^2 - n_2^2) \quad \text{s+d}_a/2 < x < \text{s+d}_a/2+d_b \]

\[ \Delta \varepsilon_a = 0 \quad \text{otherwise} \]

and

\[ \Delta \varepsilon_b = \varepsilon_0 (n_a^2 - n_2^2) \quad -d_a/2 < x < d_a/2 \]

\[ \Delta \varepsilon_b = 0 \quad \text{otherwise} \]

where we have chosen the origin of the x coordinate to be at the center of waveguide \( a \). (see p. 4)
Coupling coefficient

- Using the field distributions and the characteristic parameters of planar waveguide modes, we can calculate the relevant coefficients using
  \[
  c_{v\mu} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( \hat{E}_v^* \times \hat{H}_\mu + \hat{E}_\mu \times \hat{H}_v^* \right) \cdot \hat{z} \, dx \, dy = c_{\mu \nu}^*
  \]

- For coupling between TE modes, the only nonzero component for the electric field is \( E_y \) given in normalized form as (see Lecture 4)
  \[
  \hat{E}_y = C_{TE} \cos\left( h_1 \frac{d}{2} - \psi \right) \exp\left[ \gamma_3 (d / 2 - x) \right] \quad x > d/2
  \]
  \[
  \hat{E}_y = C_{TE} \cos(h_1x - \psi) \quad -d/2 < x < d/2
  \]
  \[
  \hat{E}_y = C_{TE} \cos(h_1d + \psi) \exp\left[ \gamma_2 (d / 2 + x) \right] \quad x < -d/2
  \]
Coupling coefficient

where

\[
\tanh_1 d = \frac{h_1(\gamma_2 + \gamma_3)}{h_1^2 - \gamma_2 \gamma_3}
\]

\[
\tan 2 \psi = \frac{h_1(\gamma_2 - \gamma_3)}{h_1^2 + \gamma_2 \gamma_3}
\]

\[
C_{TE} = \sqrt{\frac{\omega \mu_0}{\beta d_E}}
\]

\[
d_E = d + \frac{1}{\gamma_2} + \frac{1}{\gamma_3}
\]
Therefore, we have

\[ \tilde{\kappa}_{aa} = \omega \int_{-\infty}^{\infty} | \Delta \varepsilon_a | | \hat{E}_{a,y} |^2 \, dx = \frac{1}{\beta_a d_a^E} \frac{h_b^2 + \gamma_b^2}{h_a^2 + \gamma_a^2} \frac{h_a^2}{2\gamma_a} \left( 1 - e^{-2\gamma_a d_b} \right) e^{-2\gamma_a s} \]

\[ \tilde{\kappa}_{ab} = \omega \int_{-\infty}^{\infty} | \Delta \varepsilon_b \hat{E}_{a,y}^{*} \hat{E}_{b,y} | \, dx \]

\[ = \frac{1}{\sqrt{\beta_a \beta_b d_a^E d_b^E}} \sqrt{\frac{h_a^2 + \gamma_a^2}{h_b^2 + \gamma_b^2}} \frac{h_a h_b}{h_a^2 + \gamma_b^2} \left[ (\gamma_a + \gamma_b) + (\gamma_a - \gamma_b) e^{-\gamma_b d_a} \right] e^{-\gamma_b s} \]

where \( h_a \) and \( h_b \) are the parameter \( h_1 \), \( \gamma_a \) and \( \gamma_b \) are the parameter \( \gamma_2 \), and \( d_a^E \) and \( d_b^E \) are the effective waveguide thickness for the TE modes in waveguide \( a \) and waveguide \( b \).
Coupling coefficient

- The coefficient $\kappa_{bb}$ can be obtained by interchanging the indices $a$ and $b$ in the $\kappa_{aa}$ expression, while $\kappa_{ba}$ can be obtained by interchanging the indices $a$ and $b$ in the $\kappa_{ab}$ expression.
- In general $\kappa_{aa} \neq \kappa_{bb}$ and $\kappa_{ab} \neq \kappa_{ba}$.
- For TE modes, the overlap coefficient

$$c_{ab} = c_{ba}^* = \frac{\beta_a + \beta_b}{\omega \mu_0} \int_{-\infty}^{\infty} \hat{E}_{a,y}^* \hat{E}_{b,y} dx$$

$$= \frac{\beta_a + \beta_b}{\sqrt{\beta_a \beta_b d_a^E d_b^E}} \left[ \frac{h_a^2 + \gamma_a^2}{h_b^2 + \gamma_b^2} \frac{h_a h_b}{h_a^2 + \gamma_a^2} \left( \frac{1}{\gamma_a - \gamma_b} + \frac{e^{-\gamma_b d_a}}{\gamma_a + \gamma_b} \right) e^{-\gamma_b s} + \frac{h_b^2 + \gamma_b^2}{h_a^2 + \gamma_a^2} \frac{h_b h_a}{h_b^2 + \gamma_b^2} \left( \frac{1}{\gamma_b - \gamma_a} + \frac{e^{-\gamma_a d_b}}{\gamma_b + \gamma_a} \right) e^{-\gamma_a s} \right]$$

- Similar, but somewhat more complicated, formulas can be obtained for coupling between TM modes.
Coupling coefficient

- We now consider the case of a symmetric directional coupler.
- In this case, \( n_a = n_b = n_1 \) and \( d_a = d_b = d \).
- Therefore, we have \( \beta_a = \beta_b \equiv \beta \), \( h_a = h_b = h_1 \equiv h \), and \( \gamma_a = \gamma_b = \gamma_2 \equiv \gamma \), and the coefficients are much simplified.
- For coupling between TE modes of the same order, we have

\[
\tilde{\kappa}_{aa} = \tilde{\kappa}_{bb} = \frac{1}{\beta_a d_E} \frac{h^2}{2\gamma} \left(1 - e^{-2\gamma d}\right)e^{-2\gamma s}
\]

\[
\tilde{\kappa} \equiv \tilde{\kappa}_{ab} = \tilde{\kappa}_{ba}^* = \frac{2}{\beta d_E} \frac{h^2 \gamma}{h^2 + \gamma^2} e^{-\gamma s}
\]

\[
c \equiv c_{ab} = c_{ba}^* = \frac{2}{d_E} \frac{h^2}{h^2 + \gamma^2} \left(s + \frac{e^{-\gamma d}}{\gamma}\right)e^{-\gamma s}
\]

Similar formulas can be obtained for coupling between TM modes.
Coupling coefficient

- The coupling coefficient used in the coupled-mode equation is

\[ \kappa \equiv \frac{\tilde{\kappa} - c^* \tilde{\kappa}_{aa}}{1 - |c|^2} \]

\[ \beta_a + \kappa_{aa} = \beta_b + \kappa_{bb} \]

- We have \( \delta = 0 \), and the coupling is always phase matched.
Symmetric directional couplers
Symmetric directional couplers

- In an ideal symmetric directional coupler, the two waveguides are identical, and the modes are always phase matched.

- The coupling efficiency is given by

\[ \eta_{PM} = \sin^2 |\kappa| l \]

- For a desired coupling efficiency \( \eta_{PM} \), the length of the coupler has to be

\[ l = \frac{1}{\kappa} (n\pi \pm \sin^{-1} \sqrt{\eta_{PM}}) = 2 \left( n \pm \frac{1}{\pi} \sin^{-1} \sqrt{\eta_{PM}} \right) l_{c}^{PM} \]

- For complete transfer of power, the length of the coupler has to be exactly one of the odd multiples of \( l_{c} \) given in \( l = (2n + 1)l_{c}, \ n = 0, 1, 2, \ldots \)

- For a 50% coupling efficiency, we need \( l = (n + 1/2)l_{c} \), where \( n = 0, 1, 2, \ldots \), and the shortest coupler length needed is exactly \( l_{c}/2 \).
Symmetric directional couplers

- Launching optical power into one waveguide of such a coupler at its input end results in equal division of power between the two waveguides at the output end.
- The device functions as a 3-dB coupler or as a 50:50 power splitter.

- Any desired coupling efficiency can be obtained by properly choosing the length of the coupler for a given value of $\kappa$ or by choosing a proper value of $\kappa$ through the design of the coupler for a given length.

- In integrated photonic structures, changes in the coupling efficiency of a coupler can be accomplished either by altering the value of $\kappa$ or by varying the propagation constants $\beta_a$ and $\beta_b$ in the two waveguides by different amounts to introduce a finite phase mismatch $\delta$ in the coupler.
Symmetric directional couplers

- In practical devices, these changes can be created through the following:
  - *electro-optic effect*, being controllable with an externally applied voltage (or current),
  - *nonlinear optical effects*, being controllable with the optical power in the waveguides, or
  - through any other effects (e.g. thermal, mechanical strain) that cause changes in the refractive index of the medium of the coupler.

- The ability to vary the coupling efficiency of a directional coupler through a control signal results in many useful applications.
Symmetric directional couplers

- An important example is an optical switch that functions between the cross state, in which power is completely transferred from the input channel to the other channel at the output, and the parallel state (bar state), in which power is completely passed through the input channel at the output without any transfer to the other channel.
Symmetric directional couplers

- The use of a directional coupler as a *TE-TM polarization splitter*. Here, TE polarization is in the bar state, which TM polarization is in the cross state.

![Diagram showing TE+TM, TE, and TM connections in a symmetric directional coupler](image-url)
Symmetric directional couplers

- Another example is the TE-TM polarization splitter. It is possible to create a difference between the coupling coefficients, $\kappa_{EE}$ and $\kappa_{MM}$, of the TE and TM modes, even though the coupler has a symmetric structure.

- This can be accomplished by fabricating the coupler in a birefringent medium such as LiNbO$_3$ or a nonbirefringent electro-optic material such as GaAs and by applying a voltage to adjust properly the different refractive indices seen by the TE and TM fields.

- For a coupler of length $l$, polarization splitting where TE polarization is in the bar state while TM polarization is in the cross state, is attained when

$$l = \frac{n\pi}{\kappa_{EE}} = \frac{(2n \pm 1)\pi}{2\kappa_{MM}}$$

$n$ integer
Symmetric directional couplers

- This is possible if there is a difference between the coupling coefficients of the two different polarizations of the amount
  \[ \Delta \kappa \equiv |\kappa_{MM} - \kappa_{EE}| = \frac{\kappa_{EE}}{2n} \]

- For polarization splitting resulting in TE polarization in the cross state and TM polarization in the bar state, we need
  \[ \Delta \kappa = \frac{\kappa_{MM}}{2n} \]
  \[ l = \frac{n \pi}{\kappa_{MM}} \]
Grating waveguide couplers
Grating waveguide couplers

- Grating waveguide couplers have many useful applications and are one of the most important kinds of waveguide couplers.
- They consist of periodic fine structures that form gratings in waveguides.
- The grating in a waveguide can be either periodic index modulation or periodic structural corrugation.
- Periodic index modulation can be permanently written in a waveguide by periodically modulating the doping concentration in the waveguide medium, or it can be created by an electro-optic, acousto-optic, or nonlinear optical effect.
- In the latter case, the grating can be time dependent if the modulation signal is time varying. It can also be a moving grating if the modulation signal is a traveling wave.
In the case of periodic structural corrugation, the corrugation is a permanent structure of a waveguide. It is usually located at an interface between layers of different refractive indices, such as that between the guiding layer and the substrate or that between the guiding layer and the cover layer of a planar waveguide. It can also be placed away from the interfaces next to the guiding layer so long as the mode fields have sufficient penetration into the neighboring layers to see the corrugation.
Grating waveguide couplers

- The grating in a waveguide coupler can be considered as a periodic perturbation of $\Delta \varepsilon$ that has a spatial periodicity characteristic of the grating.

- In a *coplanar coupler*, the grating can have a two-dimensional periodicity while the propagation vectors of the waves being coupled are in the same plane confined by the waveguide but not necessarily parallel to each other.

- In a *collinear coupler*, the waves being coupled are propagating either codirectionally or contradirectionally, and the grating is periodic only in the propagation direction of the guided waves.

- Here we only consider the case of collinear coupling in a waveguide along the $z$ direction.
Grating waveguide couplers

The coupling coefficients are also periodic in $z$. Besides, for coupling in a single waveguide, we have $\kappa_{ab}(z) = \kappa_{ba}^*(z)$. They can be expressed in terms of the following Fourier series expansion:

$$\kappa_{ab}(z) = \omega \int \int_{-\infty}^{\infty} \hat{E}_a^* \cdot \Delta \varepsilon(x, y, z) \cdot \hat{E}_b \, dx \, dy = \sum_q \kappa_{ab}(q) \exp(iqKz)$$

and

$$\kappa_{ba}(z) = \kappa_{ab}^*(z) = \sum_q \kappa_{ab}^*(q) \exp(-iqKz)$$

where

$$K = \frac{2\pi}{\Lambda}$$

and

$$\kappa_{ab}(q) = \frac{1}{\Lambda} \int_{0}^{\Lambda} \kappa_{ab}(z) e^{-iqKz} \, dz$$
Grating waveguide couplers

- Considering the fact that efficient coupling exists only near phase matching, the coupled equations can be approximated as

\[
\pm \frac{d\tilde{A}}{dz} = i \kappa_{ab}(z) \tilde{B} e^{i \Delta \beta z} = i \tilde{B} \sum_q \kappa_{ab}(q) e^{i(\Delta \beta + qK)z} \approx i \kappa \tilde{B} e^{i(\Delta \beta + qK)z}
\]

\[
\pm \frac{d\tilde{B}}{dz} = i \kappa_{ba}(z) \tilde{A} e^{-i \Delta \beta z} = i \tilde{A} \sum_q \kappa^*_{ab}(q) e^{-i(\Delta \beta + qK)z} \approx i \kappa^* \tilde{A} e^{-i(\Delta \beta + qK)z}
\]

where the phase mismatch is given as

\[2\delta = \Delta \beta + qK = \beta_b - \beta_a + qK\]

- Only one term in the Fourier series that yields a minimum value for $|\delta|$ is kept in each of the two coupled-mode equations because only this term will effectively couple two waves. Here, we have also used $\kappa = \kappa_{ab}(q)$ for the Fourier term that is kept.
Note that though $\kappa_{ba}(z) = \kappa_{ab}^*(z)$, $\kappa_{ba}(q)$ and $\kappa_{ab}^*(q)$ are not necessarily the same unless both happen to be real quantities.

Instead, we have $\kappa_{ba}(q) = \kappa_{ab}^*(-q)$ among the Fourier components of $\kappa_{ba}(z)$ and $\kappa_{ab}^*(z)$.

The following relations hold:

$$\kappa = \kappa_{ab}(q) = \kappa_{ba}^*(-q)$$

$$\kappa^* = \kappa_{ab}^*(q) = \kappa_{ba}(-q)$$

Note that the general results obtained in two-mode coupling can be applied directly to the coupling of modes in a grating waveguide coupler with the coupling coefficients given by $\kappa$ and the phase mismatch given by $2\delta$. 

Grating waveguide couplers
Distributed Bragg reflectors
Distributed Bragg reflectors

- The function of this device is based on coupling between the forward- and backward-propagating fields of the same mode in a grating waveguide coupler.
- This is a special case of contradirectional coupling where $\beta_b = -\beta_a$ and $\Delta \beta = \beta_b - \beta_a$. In this case, we can define $\beta \equiv \beta_a = -\beta_b$
- Then, $\Delta \beta = -2\beta$
- The phase mismatch $2\delta = -2\beta + qK$
- The phase-matching condition ($2\delta = 0$) can be stated as the following Bragg condition:

$$\beta_B = \frac{qK}{2}$$

where $q$ is the integer that allows phase matching and is the \textit{order of coupling} between the two contrapropagating waves.
Distributed Bragg reflectors

- The grating period required to satisfy this phase-matching condition is

\[ \Lambda = q \frac{\pi}{\beta_B} = q \frac{\lambda_B}{2n_\beta} \]

where \( \lambda_B \) is the free-space Bragg wavelength of the field and \( n_\beta \) is the effective refractive index of the mode field in the waveguide.

- A grating with a period given above for an integer \( q \) is called the \textit{qth-order grating} for the mode coupling under consideration.

  e.g. It is a first-order grating if \( \Lambda = \lambda_B/2n_\beta \) and is a second-order grating if \( \Lambda = \lambda_B/n_\beta \).

- A simple sinusoidal grating can only be a first-order grating because \( q \) can only be 1 or -1.
Distributed Bragg reflectors

- To get an idea of the size of the grating period in a practical device structure, we consider as an example the grating in an InGaAsP waveguide for an optical wavelength of $\lambda = \lambda_B = 1.3 \, \mu m$ in free space.

- The index of refraction for InGaAsP with a bandgap energy corresponding to 1.3 $\mu m$ optical wavelength is about 3.48.

- Taking $n_\beta \approx 3.48$, we find that $\lambda_B / n_\beta \approx 374 \, nm$. We then have $\Lambda = 187 \, nm$ for a first-order grating and $\Lambda = 374 \, nm$ for a second-order grating.
Distributed Bragg reflectors

- The effect of this contradirectional coupling is an efficient transfer of power from the forward-propagating field to the backward-propagating field when $\delta^2 < |\kappa|^2$.
- From the input end of the grating waveguide coupler, it is seen that power is reflected back due to this coupling.
- This type of reflector, which relies on the coupling of waves by a distributed periodic structure, is called the distributed Bragg reflector (DBR).
- Its reflection coefficient $r$ is that given by

$$ r = |r| e^{i\phi_{\text{DBR}}} = \frac{ik_{ba} \sinh \alpha_c l}{\alpha_c \cosh \alpha_c l + i\delta \sinh \alpha_c l} $$

- Its reflectivity is simply the coupling efficiency $\eta$.

$$ \eta = \frac{\sinh^2 \left( |\kappa| l \sqrt{1 - |\delta / \kappa|^2} \right)}{\cosh^2 \left( |\kappa| l \sqrt{1 - |\delta / \kappa|^2} \right) - |\delta / \kappa|^2} $$
Surface input and output couplers
The approaches to coupling light in and out of optical waveguides, including fibers, are basically classified into two categories:

- Surface coupling
- End coupling (also called end-fire coupling)

Surface coupling relies on the coupling of an optical wave in or out of a waveguide through the longitudinal surface of the waveguide.

For end coupling, the optical wave is coupled directly through an exposed cross section at one end of the waveguide.

Here we examine the surface couplers.
Surface input and output couplers

- According to coupled-mode theory, coupling of an optical wave through the longitudinal surface of a waveguide into a guided mode is an effect of coupling between the radiation modes of the waveguide and the guided mode.

- Any unguided propagating field such as that of the incident optical wave to be coupled into the waveguide can be expanded in terms of the radiation modes of the waveguide in the form of

\[
E(r) = \sum_{\nu} A_{\nu} \hat{E}_{\nu}(x, y) \exp(i \beta_{\nu} z)
\]

\[
H(r) = \sum_{\nu} A_{\nu} \hat{H}_{\nu}(x, y) \exp(i \beta_{\nu} z)
\]

with the summation replaced by integration over all radiation modes.
Surface input and output couplers

- For efficient coupling, the phase-matching condition has to be satisfied.
- Phase matching is not possible, however, without some special arrangements to perturb the system because the longitudinal propagation constant of any radiation mode is always smaller than that of a guided mode.
- For a beam incident on the surface of the guiding layer at an angle $\theta$, the largest propagation constant is that of a plane wave, $k = k_3$.
- Its longitudinal propagation constant is the $z$ component, which satisfies

$$k_z = k_3 \sin \theta < k_3 < \beta$$

where $\beta$ is the propagation constant of a guided mode.
Surface input and output couplers

- By the same argument and by the reciprocity theorem for electromagnetic waves, it is equally impossible to couple the field in a guided mode out of the waveguide through the surface of the waveguide without some special arrangements.

- The task of a surface coupler is therefore to change this situation and to attain phase matching s.t. a radiation field can be coupled to the guided mode.

- The same coupler can be used as an output coupler to couple a guided field out to a radiation field.
Prism couplers

- One approach to attaining phase matching is to use a prism of high index of refraction, $n_p$, as a surface coupler.
- The cover of the waveguide is usually air or some low-index fluid filling the gap $s$ between the prism and the waveguide core.

Note that $\theta$ is measured inside the prism. It is necessary that total internal reflection occurs for the incident field inside the prism in order to enable the *evanescent coupling*.
Prism couplers

- For $k_p = n_p \omega / c$, the phase-matching condition is then
  \[ k_p \sin \theta = \beta \]
  which can be attained if $n_p > n_1 > n_3$ and
  \[ \theta > \theta_c = \sin^{-1}(n_3 / n_p) \]

- As a result, the field does not propagate freely in the gap between the prism and the waveguide core. The total internal reflection results in an exponentially decaying evanescent field in the gap between the prism and the waveguide. Coupling to the waveguide mode occurs through optical tunneling (evanescent coupling) when this evanescent field overlaps with the field of a guided mode if the phase-matching condition is satisfied.
Grating couplers

- Another approach to obtaining phase matching for the coupling between the radiation field and a guided mode is the use of a grating.
- Input and output coupling of a planar waveguide can be attained using a grating surface coupler.
- The function of the grating is to provide an extra phase factor $qK$ in a manner similar to that of the grating waveguide couplers so that modes of different propagation constants can be phase matched and efficiently coupled.
- However, because the radiation fields are not restricted to a single propagation direction, the situation here is more complicated than that of the coupling between guided modes.
- The radiation fields can exist simultaneously in both the cover and the substrate regions, and the grating can scatter the light into different diffraction orders.
Grating couplers

- Because the grating is periodic only along the z direction, the extra phase factor $qK$ it provides is also only in the z direction.
- Therefore, the phase-matching condition for a guided mode with a propagation constant $\beta$ is

$$k_{iq,z} + qK = \beta, \ i = 2, 3$$

Or

$$k_i \sin \theta_{iq} + qK = \beta, \ i = 2, 3$$

where $i = 2$ for the radiation fields in the substrate region, $i = 3$ for the radiation field in the cover region.
- Because $k_1 > \beta > k_2 > k_3$, phase matching is possible only for $q \geq 1$.